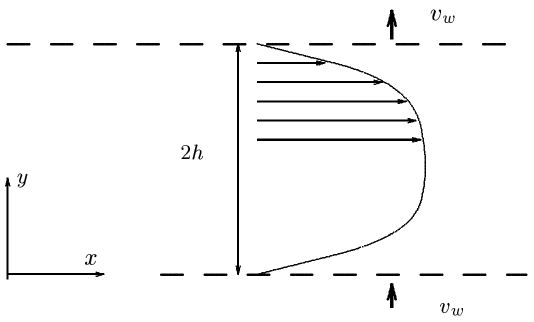

FIGURE 1. Schematic of the plane channel flow configuration with streamwise constant cross-flow.

1. Fully developed laminar flow in a plane channel with constant cross flow

We consider fully developed laminar flow in a plane channel (figure 1). In this case, all variables are constant in time and in the spanwise direction and it is sufficient to consider a one-dimensional problem in the wall-normal direction y. The streamwise momentum and continuity read:

$$v\,\partial_y\,u + \partial_x\,p = \nu\,\partial_{yy}\,u \tag{1.1}$$

$$\partial_y\,v = 0 \quad . \tag{1.2}$$

The cross-flow is therefore constant, $v = v_w$, and we can integrate (1.1) subject to the non-slip condition $u(0, 2h) = 0$, viz.

$$\frac{u(y)}{v_w} = \frac{\partial_x p}{v_w^2\,h}\left(y/h(1 - e^{2\,Re}) - 2\,(1 - e^{Re\,y/h})\right) \quad , \tag{1.3}$$

where all variables are non-dimensionalized by $v_w$and h and the Reynolds number of the cross-flow reads $Re = v_w\,h/\nu$. We calculate the bulk velocity $u_b = \int_0^{2h} u\mathrm{d}y/2h$,

$$u_b = \partial_x\,p\frac{(-1 - e^{2\,Re}) + \frac{1}{Re}(e^{2\,Re} - 1)}{e^{2\,Re} - 1} \quad . \tag{1.4}$$

The pressure gradient can thus be expressed in terms of the bulk velocity, viz.

$$\partial_x\,p = u_b\frac{e^{2\,Re} - 1}{-1 - e^{2\,Re} - \frac{1}{Re}(e^{2\,Re} - 1)} \quad . \tag{1.5}$$

The velocity gradient at the two walls, relative to the gradient when the cross-flow is zero, $\partial_y u = 3u_b$, now becomes:

$$\frac{c_f}{c_f(v_w = 0)} = \begin{cases} -\dfrac{1}{3}\dfrac{e^{2\,Re} - 1 - 2\,Re}{-1 - e^{2\,Re} + \frac{1}{Re}(e^{2\,Re} - 1)} & (y = 0) \\ -\dfrac{1}{3}\dfrac{e^{2\,Re} - 1 - 2\,Re\,e^{2\,Re}}{-1 - e^{2\,Re} + \frac{1}{Re}(e^{2\,Re} - 1)} & (y = 2h) \end{cases} \quad . \tag{1.6}$$

Figure 2 shows the relative skin friction factor as a function of the cross-flow Reynolds number. It can be seen that the friction increases more rapidly on the suction side than it decreases on the injection side. Figure 3 shows that the sum of the wall friction of both walls increases for any finite cross-flow rate.

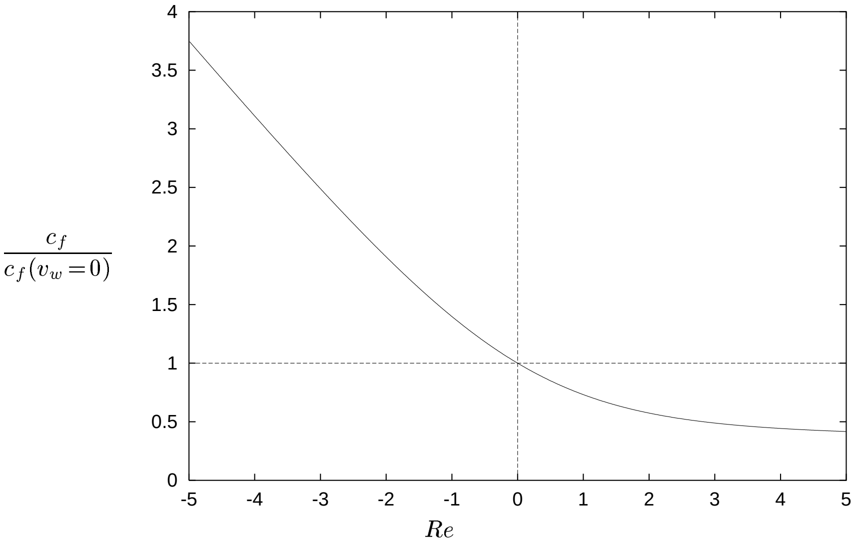

FIGURE 2. Relative skin friction as a function of the Reynolds number of the wall-normal velocity.

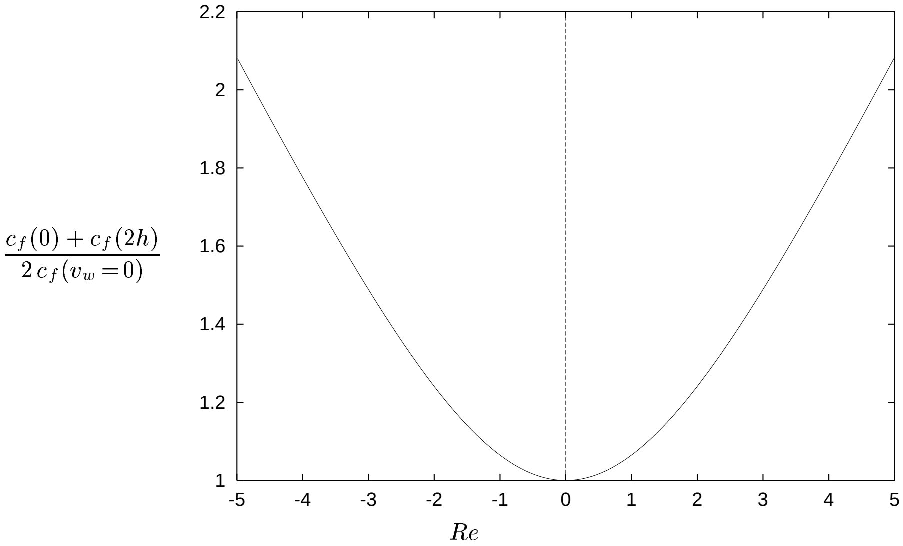

FIGURE 3. Mean skin friction of the channel with constant cross-flow as a function of the Reynolds number of the wall-normal velocity.