1. Linear stability analysis of channel flow with turbulent viscosity

Perhaps we should take into account the action of turbulence upon the perturbations in our stability analysis of a plane channel with one porous wall. We can consider an analysis alike Reynolds (1972) who carries an algebraic expression for the unknown stresses. The velocity field is divided into three parts, $u = \overline{u} + \widetilde{u} + u'$: the time average $\overline{u}$, the coherent perturbation of normal mode type $\widetilde{u}$ and the turbulent fluctuation u'. A phase average (following the phase of the coherent perturbation) can be defined as $< u >= \overline{u} + \widetilde{u}$. Substituting the decomposition into the momentum equations, taking the phase average and subtracting the time-mean leads to the following equation, where the non-linearity of $\widetilde{u}$ has been neglected:

$$\partial_t \widetilde{u}_i + \overline{u}_{i,j} \cdot \widetilde{u}_j + \overline{u}_j \cdot \widetilde{u}_{i,j} + \left(< u'_j u'_i > - \overline{u_i{}' u'_j} \right)_{,j} + \widetilde{p}_{,i} = \nu \nabla^2 \widetilde{u}_i \quad . \tag{1.1}$$

The correlation term

$$r_{ij} \equiv < u'_j u'_i > - \overline{u'_i u'_j} = \widetilde{u'_i u'_j} \tag{1.2}$$

corresponds to the Reynolds stress due to the wave-like perturbations that we want to investigate and it has to be modeled. As in Reynolds (1972) we will use Boussinesq's constitutive relation in order to convert the unknown into a linear term:

$$r_{ij} = -2\,\nu_t\,\widetilde{S}_{ij} \qquad \widetilde{S}_{ij} = \frac{1}{2}\left(\widetilde{u}_{i,j} + \widetilde{u}_{j,i}\right) = \frac{1}{2}\begin{bmatrix} 2\,\psi_{,xy} & \nabla^2\psi \\ \nabla^2\psi & -2\,\psi_{,xy} \end{bmatrix} \quad , \tag{1.3}$$

where the eddy viscosity ν_t is isotropic. However, we will not suppose a constant eddy viscosity due to the inhomogeneous character of turbulence in the wall-normal direction of the channel, i.e. $\nu_t = \nu_t(y)$. The analogue of the Orr-Sommerfeld equation now reads for the normal mode coefficients of the perturbation streamfunction $\psi = \hat{\psi}\, e^{i\alpha(x-ct)}$:

$$\begin{aligned} &\left[(\overline{u} - c)(\partial_{yy} - \alpha^2) - \frac{\nu}{i\alpha}(\partial_{yy} - \alpha)^2 - (\partial_{yy}\overline{u})\right] \hat{\psi} + \\ &\frac{1}{i}\left[-\alpha\nu_t(\partial_{yy} - \alpha^2)\hat{\psi} + 4\,\alpha\,\partial_y(\nu_t\,\partial_y\,\hat{\psi}) - \frac{1}{\alpha}\partial_{yy}(\nu_t\,(\partial_{yy} - \alpha^2)\,\hat{\psi})\right] \quad = \quad 0 \end{aligned} \quad . \tag{1.4}$$

The terms in the second square bracket are due to the background turbulence. The boundary conditions for the Reynolds-averaged problem are the following:

$$\begin{aligned} &\hat{\psi} = 0\,, && \partial_y\hat{\psi} = 0 \qquad (y = 2h) \\ &\beta\,\hat{\psi}\,\partial_y\overline{u} - i\alpha\hat{\psi} + \frac{\beta\,\nu}{i\alpha}\partial_{yyy}\hat{\psi} - \frac{\beta}{i\,\alpha}\partial_y\left(\nu_t(\partial_{yy}\hat{\psi} + \alpha^2\hat{\psi})\right) = 0\,, && \partial_y\hat{\psi} = 0 \qquad (y = 0)\,. \end{aligned} \tag{1.5}$$

The contribution of turbulent viscosity vanishes, however, since we use a profile that verifies $\nu_t(0) = \partial_y\nu_t(0) = 0$.

After applying the usual B-spline Galerkin method, we are left with seven new eleven-banded matrices of basic dimension N_s (N_s is the number of splines that is employed to span the wall-normal direction) to assemble. Note that the use of the general eigenvalue solver `ZGEGV` from LAPACK leads to non-physical results in this context and that we instead resort to routines from the EISPACK library.

The variation of eddy viscosity is extracted from direct simulation data of plane channel flow by using the definition $\nu_t = -\overline{u'v'}/\overline{u}_{,y}$. This expression is ill-behaved near the centerline of the channel, where the mean gradient vanishes. Figure 1 shows the profile of this viscosity taken from case 16 of Jiménez, Pinelli & Uhlmann (1998*b*), smoothed in the center and symmetrized with respect to $y = h$ by taking the mean of both channel halves.

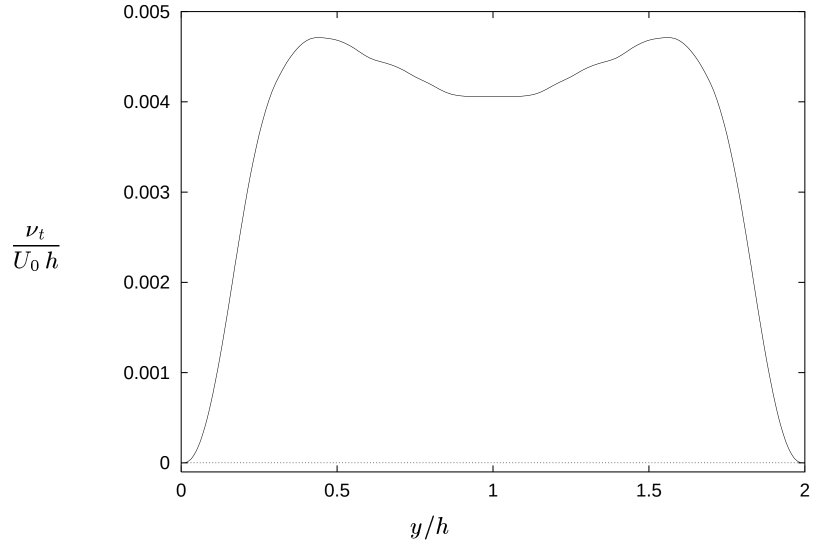

FIGURE 1. Variation of turbulent viscosity from case 16 of Jiménez *et al.* (1998*b*). The curve has been smoothed in the center and symmetrized with respect to $y=h$.

Since turbulent viscosity is low in the immediate wall-region, we expect that the inclusion of the additional terms does not substantially alter our previous results (Jiménez, Kawahara, Pinelli & Uhlmann (1998*a*)). Figure 2 shows that the difference due to the inclusion of turbulent viscosity in our analysis is in fact small. A slight stabilization of the perturbations is noted when the eddy viscosity is taken into account. The corresponding most unstable eigenfunctions (not shown) are equally quite insensitive to the inclusion of turbulent viscosity.

2. Outlook

Would it be a good idea to verify this issue in the case of three-dimensional streak stability analysis?

REFERENCES

JIMÉNEZ, J., KAWAHARA, G., PINELLI, A. & UHLMANN, M. 1998*a* Linear stability analysis of turbulent channel flow including porous walls. Technical Note ETSIA MF-9811, School of Aeronautics, Universidad Politécnica Madrid, Spain.

JIMÉNEZ, J., PINELLI, A. & UHLMANN, M. 1998*b* Plane channel flow simulation over porous walls. Technical Note ETSIA MF-9809, School of Aeronautics, Universidad Politécnica Madrid, Spain.

REYNOLDS, W. 1972 Large-scale instabilities of turbulent wakes. *J. Fluid Mech.* **54** (3), 481–488.

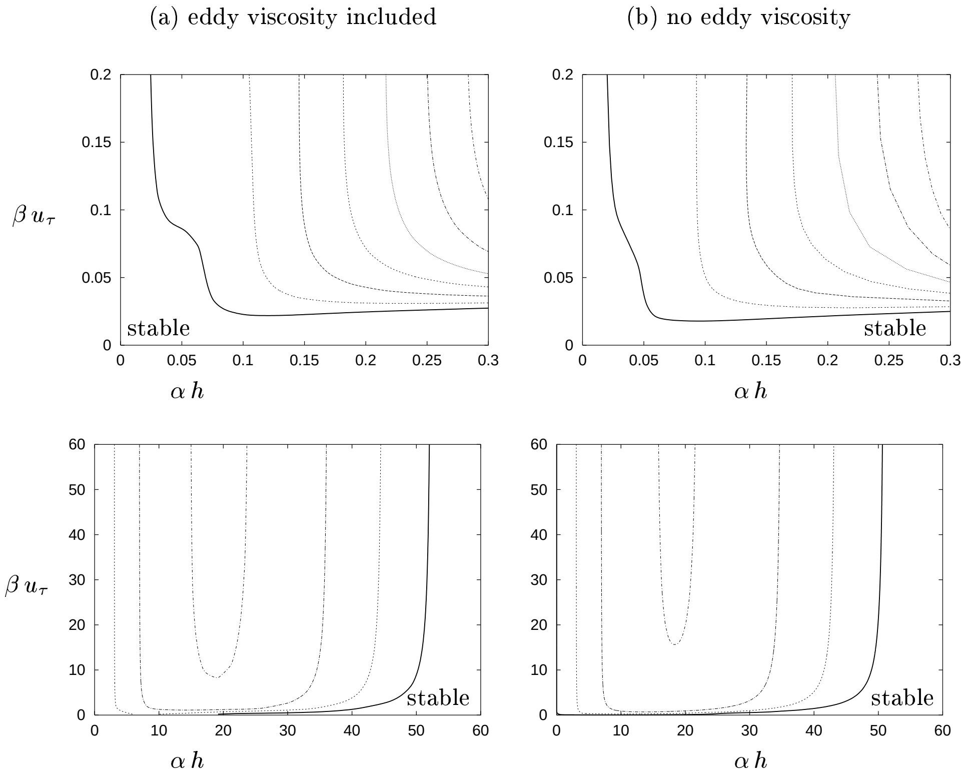

FIGURE 2. Isolines of the growth rate of normal-mode perturbations of an equilibrium turbulent channel flow profile with one porous wall; data is represented as a function of streamwise wavenumber α of the perturbation and porosity coefficient β. The thick solid line indicates the neutral curve. Contour-levels are $0\,(1)\,6$ in the upper row and $0\,(10)\,30$ in the lower row.